Deriválás

Alapderiváltak

Elnevezés	Szabály	Példák
D1	$(c)' = 0$	$(5)' = 0$
		$(\cos\pi)' = 0$
		$(\ln 5)' = 0$
		$(e^2)' = 0$
D2	$(x^n)' = nx^{n-1}$	$(x^8)' = 8x^7$
		$(\sqrt{x})' = \left(x^{\frac{1}{2}}\right)' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \mathbf{\frac{1}{2\sqrt{x}}}$
		$(\sqrt[6]{x})' = \left(x^{\frac{1}{6}}\right)' = \frac{1}{6}x^{-\frac{5}{6}} = \frac{1}{6x^{\frac{5}{6}}} = \mathbf{\frac{1}{2\sqrt[6]{x^5}}}$
		$\left(\sqrt[7]{x^3}\right)' = \left(x^{\frac{3}{7}}\right)' = \frac{3}{7}x^{-\frac{4}{7}} = \frac{3}{7x^{\frac{4}{7}}} = \mathbf{\frac{3}{7\sqrt[7]{x^4}}}$
		$\left(\frac{1}{x}\right)' = (x^{-1})' = -x^{-2} = -\mathbf{\frac{1}{x^2}}$
		$\left(\frac{1}{x^4}\right)' = (x^{-4})' = -4x^{-5} = -\mathbf{\frac{4}{x^5}}$
		$\left(\frac{1}{\sqrt{x}}\right)' = \left(\frac{1}{x^{\frac{1}{2}}}\right)' = \left(x^{-\frac{1}{2}}\right)' = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2x^{\frac{3}{2}}} = -\mathbf{\frac{1}{2\sqrt{x^3}}}$
		$\left(\frac{1}{\sqrt[3]{x}}\right)' = \left(\frac{1}{x^{\frac{1}{3}}}\right)' = \left(x^{-\frac{1}{3}}\right)' = -\frac{1}{3}x^{-\frac{4}{3}} = -\frac{1}{3x^{\frac{4}{3}}} = -\mathbf{\frac{1}{3\sqrt[3]{x^4}}}$
		$\left(\frac{1}{\sqrt[5]{x^9}}\right)' = \left(\frac{1}{x^{\frac{9}{5}}}\right)' = \left(x^{-\frac{9}{5}}\right)' = -\frac{9}{5}x^{-\frac{14}{5}} = -\frac{9}{5x^{\frac{14}{5}}} = -\mathbf{\frac{9}{5\sqrt[5]{x^{14}}}}$
D3	$(x)' = 1$	$(x)' = 1$
D4	$(e^x)' = e^x$	$(e^x)' = e^x$
D5	$(a^x)' = a^x \cdot \ln a$	$(3^x)' = 3^x \cdot \ln 3$
D6	$(\ln x)' = \frac{1}{x}$	$(\ln x)' = \frac{1}{x}$
D7	$(\log_a x)' = \frac{1}{x} \cdot \frac{1}{\ln a}$	$(\log_6 x)' = \frac{1}{x} \cdot \frac{1}{\ln 6}\left(= \frac{1}{x \cdot \ln 6}\right)$
		$(\lg x)' = (\log_{10} x)' = \frac{1}{x} \cdot \frac{1}{\ln 10}\left(= \frac{1}{x \cdot \ln 10}\right)$
D8	$(\sin x)' = \cos x$	$(\sin x)' = \cos x$
D9	$(\cos x)' = -\sin x$	$(\cos x)' = -\sin x$

Elnevezés	Szabály	Példák
D10	$(tg\,x)' = \frac{1}{cos^2 x}$	$(tg\,x)' = \frac{1}{cos^2 x}$
D11	$(ctg\,x)' = -\frac{1}{sin^2 x}$	$(ctg\,x)' = -\frac{1}{sin^2 x}$
D12	$(sh\,x)' = ch\,x$	$(sh\,x)' = ch\,x$
D13	$(ch\,x)' = sh\,x$	$(ch\,x)' = sh\,x$
D14	$(th\,x)' = \frac{1}{ch^2 x}$	$(th\,x)' = \frac{1}{ch^2 x}$
D15	$(cth\,x)' = -\frac{1}{sh^2 x}$	$(cth\,x)' = -\frac{1}{sh^2 x}$
D16	$(arcsin\,x)' = \frac{1}{\sqrt{1-x^2}}$	$(arcsin\,x)' = \frac{1}{\sqrt{1-x^2}}$
D17	$(arccos\,x)' = -\frac{1}{\sqrt{1-x^2}}$	$(arccos\,x)' = -\frac{1}{\sqrt{1-x^2}}$
D18	$(arctg\,x)' = \frac{1}{1+x^2}$	$(arctg\,x)' = \frac{1}{1+x^2}$
D19	$(arcctg\,x)' = -\frac{1}{1+x^2}$	$(arcctg\,x)' = -\frac{1}{1+x^2}$
D20	$(arsh\,x)' = \frac{1}{\sqrt{x^2+1}}$	$(arsh\,x)' = \frac{1}{\sqrt{x^2+1}}$
D21	$(arch\,x)' = \frac{1}{\sqrt{x^2-1}}$	$(arch\,x)' = \frac{1}{\sqrt{x^2-1}}$
D22	$(arth\,x)' = \frac{1}{1-x^2}$	$(arth\,x)' = \frac{1}{1-x^2}$

Deriválási szabályok

Elnevezés	Szabály	Példák
DSZ1	$(c \cdot f)' = c \cdot f'$	$(8x^6)' = \mathbf{8} \cdot \mathbf{6x^5} = \mathbf{48x^5}$
DSZ2	$\left(\frac{f}{c}\right)' = \frac{f'}{c}$	$\left(\frac{x^4}{6}\right)' = \frac{\mathbf{4x^3}}{\mathbf{6}} = \frac{\mathbf{2}}{\mathbf{3}}\mathbf{x^3}$
DSZ3	$(f \pm g)' = f' \pm g'$	$(x^3 + 7^x)' = \mathbf{3x^2} + \mathbf{7^x \cdot \ln 7}$
		$(\sin x - \ln x)' = \mathbf{\cos x} - \frac{\mathbf{1}}{\mathbf{x}}$
DSZ4	$(f \cdot g)' = f' \cdot g + f \cdot g'$	$(x^4 \cos x)' = \mathbf{4x^3} \cdot \mathbf{\cos x} + \mathbf{x^4} \cdot (\mathbf{-\sin x})$
DSZ5	$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$	$\left(\frac{\sin x}{x^3}\right)' = \frac{\mathbf{\cos(x)} \cdot \mathbf{x^3} - \mathbf{\sin(x)} \cdot \mathbf{3x^2}}{(\mathbf{x^3})^2}$
DSZ6	$(f(g(x)))' = f'(g(x)) \cdot g'(x)$	$(\sin(x^5))' = \mathbf{\cos(x^5)} \cdot \mathbf{5x^4}$
		$(7^{\cos x})' = \mathbf{7^{\cos x} \cdot \ln 7} \cdot (\mathbf{-\sin x})$

DSZ7	$(f^g)' = \left(e^{\ln f^g}\right)' = (e^{g \cdot \ln f})' = e^{g \cdot \ln f}(g' \cdot \ln f + g \cdot (\ln f)') = f^g \cdot (g' \cdot \ln f + g \cdot (\ln f)')$
Példa 1	$(x^x)' = \left(e^{\ln x^x}\right)' = (e^{x \cdot \ln x})' = e^{x \cdot \ln x} \cdot (1 \cdot \ln x + x \cdot (\ln x)') =$ $= x^x \cdot \left(1 \cdot \ln x + x \cdot \frac{1}{x}\right) = \mathbf{x^x \cdot (\ln x + 1)}$
Példa 2	$((\sin x)^{\cos x})' = \left(e^{\ln(\sin x)^{\cos x}}\right)' = \left(e^{\cos x \cdot \ln(\sin x)}\right)' =$ $= e^{\cos x \ln(\sin x)} \cdot (-\sin x \cdot \ln(\sin x) + \cos x \cdot (\ln(\sin x))') =$ $= (\sin x)^{\cos x} \cdot \left(-\sin x \cdot \ln(\sin x) + \cos x \cdot \frac{1}{\sin x} \cdot \cos x\right) =$ $= \mathbf{(\sin x)^{\cos x} \cdot \left(-\sin x \cdot \ln(\sin x) + \frac{\cos^2 x}{\sin x}\right)}$